

Quantum Vacuum and Inertial Reaction in Nonrelativistic QED

Hiroki Sunahata¹, Alfonso Rueda² * and Bernard Haisch³

¹ Dept. Physics and Astronomy, California State Univ., Long Beach, CA 90840

² Dept. Electrical Engineering, California State Univ., Long Beach, CA 90840 email: arueda@csulb.edu

³ Calphysics Inst., 519 Cringle Drive, Redwood Shores, CA 94065 email: astro@calphysics.org

Abstract

The possible connection between the electromagnetic zero-point field (ZPF) and the inertia reaction force was first pointed out by Haisch, Rueda, and Puthoff (*Phys. Rev. A*, **49**, 678, 1994), and then by Rueda and Haisch following a totally different and more satisfactory approach (*Found. of Phys.*, **28**, 1057, 1998; *Phys. Letters A*, **240**, 115, 1998; *Annalen der Physik* **10**(5), 393, 2001). In the present paper, the approach taken by Rueda and Haisch will be followed, but the analysis will be done within a formulation that uses nonrelativistic quantum electrodynamics with the creation and annihilation operators rather than the approach of Rueda and Haisch using stochastic electrodynamics. We analyze the interaction between the zero-point field and an object under hyperbolic motion (constant proper acceleration), and find that there arises a reaction force which is proportional in magnitude, and opposite in direction, to the acceleration. This is suggestive of what we know as inertia. We also point out that the equivalence principle – that inertial mass and gravitational mass have the same values – follows naturally using this approach. Inertial mass and gravitational mass are not merely equal, they are the identical thing viewed from two complementary perspectives (*Annalen der Physik* **14**(8), 479, 2005). In the first case an object accelerating through the electromagnetic zero-point field experiences resistance from the field. In the case of an object held fixed in a gravitational field, the electromagnetic zero-point field propagates on curved geodesics, in effect accelerating with respect to the fixed object, thereby generating weight. Hence, the equivalence principle does not need to be independently postulated.

Keywords: quantum vacuum, mass, zero-point field, inertia, gravitation, stochastic electrodynamics

1. Introduction

The so-called zero-point field (ZPF) is a random electromagnetic field that exists even at the temperature of absolute zero. The existence of this field first came to be known through the study of the blackbody radiation spectrum early in the twentieth century [1], and became gradually better understood with the advance of quantum theory. Moreover, the developments of Stochastic Electrodynamics (SED) in the last decades of the twentieth century have expanded its boundary and found new applications. Rueda, Haisch and Puthoff proposed that the origin of inertia could be explained, in part, as due to the interaction between an accelerated object and the zero-point-vacuum-fields. (Only the electromagnetic contribution has been studied so far from this viewpoint.) In their first approach [2], the Lorentz force that the electromagnetic zero-point field (ZPF) exerts upon an accelerating harmonic oscillator was calculated, and in the second by Rueda and Haisch [3], a more general method was taken by analyzing the zero-point-field Poynting vector that an extended accelerating object sweeps through. In this paper, the second method will be followed using a *Quantum Electrodynamics* (QED) approach for all the averaging calculations. We use the low energy version of QED, also called nonrelativistic QED [4]. It will be shown that the same results reported in Rueda and Haisch [3] are obtained with QED as well: There is a contribution to the inertia reaction force coming from the electromagnetic quantum vacuum. Other contributions are naturally expected from the other quantum vacuum fields manifested in the so called physical vacuum when it is taken as a medium. We call this the *quantum vacuum inertia hypothesis*.

The general objective of this research program is and has been to elucidate the mechanism behind the appearance of an inertial reaction force when a macroscopic body is being accelerated by an external agent. There are several contributing factors that independently contribute but so far only the electromagnetic vacuum contribution has been preliminarily explored. This contribution, although relatively minor, should display some common features that we expect will help uncover the mechanism behind the contributions from

* Send correspondence to this author.

other vacua, namely those of the strong and the weak interactions. For example, it is ordinarily expected that the gluonic fields of the strong interaction should give a major contribution to the mass of macroscopic bodies. Though the *how* of the corresponding part of the inertial reaction due to the gluonic fields is not well understood, it is expected that elucidating the mechanism for the electromagnetic field contribution will help in finding the corresponding mechanism for the more involved gluonic field.

There are however some common misconceptions as to the manner in which the inertial reaction force appears. There is, for example, the idea that the vacuum inertia hypothesis implies some ram-pressure-like action by the vacuum fields on the accelerated body. The analysis of [3] does not support such views.

In this research it is found that the electromagnetic vacuum exerts a peculiar opposition to any change in the change of a body comoving inertial frame. This opposition is in a direction opposite to that of the acceleration and of a magnitude proportional to that acceleration. And, very importantly, it is instantaneously applied to all points within the body. So this means that the opposition is not coming from the outside as a kind of wind or ram-pressure. It is easy to see that the shape of a body or its relative orientation with regard to the acceleration vector makes no difference whatsoever.

In section 2, we review the similarities and differences between the SED and QED formulations. In section 3, using a QED formulation, the inertia reaction force is obtained. We find that there exists an interaction between the ZPF and an accelerating object, and that the reaction force is proportional to the acceleration in magnitude, but opposite to the acceleration in direction. The vacuum seems to react against accelerating objects in a manner reminiscent of what is understood as inertia.

2. Comparison between the Quantum and the Stochastic Formalism

We proceed from a comparison between the quantum formalism of QED and the classical formalism of SED. The *classical* electromagnetic zero-point radiation can be written, as a superposition of plane waves [5]:

$$\mathbf{E}(\mathbf{r}, t) = \sum_{\lambda=1}^2 \int d^3k \hat{\epsilon}(\mathbf{k}, \lambda) h_{zp}(\omega) \cos[\mathbf{k} \cdot \mathbf{r} - \omega t - \theta(\mathbf{k}, \lambda)] , \quad (1)$$

$$\mathbf{B}(\mathbf{r}, t) = \sum_{\lambda=1}^2 \int d^3k (\hat{k} \times \hat{\epsilon}) h_{zp}(\omega) \cos[\mathbf{k} \cdot \mathbf{r} - \omega t - \theta(\mathbf{k}, \lambda)] . \quad (2)$$

Here, the zero-point radiation is expressed in expansion of plane waves and as a sum over two polarization states $\hat{\epsilon}(\mathbf{k}, \lambda)$, which is a function of the propagation vector \mathbf{k} and the polarization index $\lambda = 1, 2$.

From now on and in the above, the polarization components $\hat{\epsilon}_i(\mathbf{k}, \lambda)$ are to be understood as scalars. They are the projections of the polarization unit vectors $\hat{\epsilon}(\mathbf{k}, \lambda)$ onto the i -axis: $\hat{\epsilon}_i(\mathbf{k}, \lambda) = \hat{\epsilon} \cdot \hat{x}_i$. We also use the same notational convention with the \hat{k} unit vector, i.e., $\hat{k}_x = \hat{k} \cdot \hat{x}$.

In the expressions (1) and (2), the random phase $\theta(\mathbf{k}, \lambda)$ is introduced, following Planck [6], and Einstein and Hopf [7], to generate the random, fluctuating nature of the radiation. This $\theta(\mathbf{k}, \lambda)$ is a random variable distributed uniformly in the interval $(0, 2\pi)$ and independently for each wave vector \mathbf{k} and the polarization index λ . Also the spectral function $h_{zp}(\omega)$ is introduced to set the magnitude of the zero-point radiation, which is found in terms of the Planck's constant h as $h_{zp}^2(\omega) = \hbar\omega/2\pi^2$ [5]. Planck's constant enters the theory at this point only as a scale factor to attain correspondence between zero temperature random radiation of (classical) stochastic electrodynamics and the vacuum zero point field of quantum electrodynamics.

The QED formulation of the zero point fields are also expressed by the expansion in plane waves as [8,9]

$$\bar{\mathbf{E}}(\mathbf{r}, t) = \sum_{\lambda=1}^2 \int d^3k \hat{\epsilon}(\mathbf{k}, \lambda) H_{zp}(\omega) [\alpha(\mathbf{k}, \lambda) \exp(i\Theta) + \alpha^\dagger(\mathbf{k}, \lambda) \exp(-i\Theta)] , \quad (3)$$

$$\bar{\mathbf{B}}(\mathbf{r}, t) = \sum_{\lambda=1}^2 \int d^3k (\hat{k} \times \hat{\epsilon}) H_{zp}(\omega) [\alpha(\mathbf{k}, \lambda) \exp(i\Theta) + \alpha^\dagger(\mathbf{k}, \lambda) \exp(-i\Theta)] , \quad (4)$$

where $\Theta = \mathbf{k} \cdot \mathbf{r} - \omega t$.

We notice here that the cosine functions used in the SED formulation are now replaced by the exponential functions and the quantum operators $\alpha(\mathbf{k}, \lambda)$ and $\alpha^\dagger(\mathbf{k}, \lambda)$. These annihilation and creation operators have the expectation values:

$$\langle 0 | \alpha(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') | 0 \rangle = \delta_{\lambda, \lambda'} \delta^3(\mathbf{k} - \mathbf{k}') , \quad (5a)$$

$$\langle 0 | \alpha^\dagger(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') | 0 \rangle = \langle 0 | \alpha(\mathbf{k}, \lambda) \alpha(\mathbf{k}', \lambda') | 0 \rangle = \langle 0 | \alpha^\dagger(\mathbf{k}, \lambda) \alpha(\mathbf{k}', \lambda') | 0 \rangle = 0 . \quad (5b)$$

The overline on E and B in Eq. (3) and (4) indicates that these fields are now given as operators, and the spectral function in QED is $H_{zp}^2(\omega) = \hbar\omega/4\pi^2$ [10], which differs, by a small factor, from the corresponding SED spectral function we used in previous papers [3].

It is well known that only in a certain limited set of cases do SED and QED give the same results [11]. Almost forty years ago, T. H. Boyer [8] presented a detailed comparison between these two theories for the case of *free electromagnetic fields* and for *dipole oscillator systems*. In his comparison, it was found that if the QED operators are symmetrized (written in symmetric order), then the stochastic averaging of SED and the quantum averaging of QED yield exactly the same results.

This last point is of much importance for our developments. The SED stochastic averaging over the random phases yields for the electric field autocorrelation function, at two different space-time locations (\mathbf{r}_1, t_1) and (\mathbf{r}_2, t_2) , an expression of the form

$$\langle E_i(\mathbf{r}_1, t_1) E_j(\mathbf{r}_2, t_2) \rangle = \int d^3k (\delta_{ij} - \hat{k}_i \hat{k}_j) \frac{\hbar\omega}{4\pi^2} \cos[\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2) - \omega(t_1 - t_2)] , \quad (6)$$

where the subindices i and j stand for any two different cartesian space directions, $i, j = x, y, z$ and the $\langle \dots \rangle$ parentheses mean a stochastic averaging. On the other hand, if we do a simple quantum averaging over the vacuum field we get

$$\langle 0 | \overline{E}_i(\mathbf{r}_1, t_1) \overline{E}_j(\mathbf{r}_2, t_2) | 0 \rangle = \int d^3k (\delta_{ij} - \hat{k}_i \hat{k}_j) \frac{\hbar\omega}{4\pi^2} \exp[i\mathbf{k} \cdot (\mathbf{r}_1 - \mathbf{r}_2) - i\omega(t_1 - t_2)] , \quad (7)$$

which clearly shows that $\langle \dots \rangle$ of Eq. (6) and $\langle 0 | \dots | 0 \rangle$ of Eq. (7) are *not the same*. This is of course not surprising. The stochastic averaging of Eq. (6) involves averaging over the random phases in a manner thoroughly described in Refs. [2,3,8,11,12,13]. On the other hand, the averaging described in Eq. (7) is the standard quantum averaging where the \overline{E}_i and \overline{E}_j fields are taken as operators in the Heisenberg picture. Nevertheless, if instead of writing the operator fields as in Eq. (7), we write them in terms of a symmetrized expression, then we have that

$$\begin{aligned} \langle E_i(\mathbf{r}_1, t_1) E_j(\mathbf{r}_2, t_2) \rangle &= \frac{1}{2} \left[\langle 0 | \overline{E}_i(\mathbf{r}_1, t_1) \overline{E}_j(\mathbf{r}_2, t_2) | 0 \rangle + \langle 0 | \overline{E}_j(\mathbf{r}_2, t_2) \overline{E}_i(\mathbf{r}_1, t_1) | 0 \rangle \right] \\ &= \left\langle 0 \left| \frac{\overline{E}_i(\mathbf{r}_1, t_1) \overline{E}_j(\mathbf{r}_2, t_2) + \overline{E}_j(\mathbf{r}_2, t_2) \overline{E}_i(\mathbf{r}_1, t_1)}{2} \right| 0 \right\rangle . \end{aligned} \quad (8)$$

In Refs. [3,10], correlations of the form $\langle 0 | \overline{E}_i(\mathbf{r}_1, t_1) \overline{B}_j(\mathbf{r}_2, t_2) | 0 \rangle$ were calculated in an effort to evaluate $\langle 0 | \overline{\mathbf{E}} \times \overline{\mathbf{B}} | 0 \rangle$. In this case as well, if the quantum operators are properly symmetrized, the stochastic averaging of SED and the quantum averaging of QED give identical results. See Appendix A for more details.

3. Origin of the Electromagnetic Vacuum Contribution to the Inertia Reaction Force

Let us consider an object to be uniformly accelerated by a force applied to it by an external agent and such that the object moves rectilinearly along the x -axis with constant proper acceleration $\mathbf{a} = \hat{x}a$. We need only look at the coordinates of the center of mass and for most purposes view the object as point-like. The object then performs so-called hyperbolic motion [14,15]. Assume the body was instantaneously at rest at time $t_* = 0$ in an inertial frame I_* that we call the laboratory frame. Consider a non-inertial frame S such that its x -axis coincides with that of I_* and let the body be located at coordinates $(c^2/a, 0, 0)$ in S at all times.

So this point of S performs hyperbolic motion. The acceleration of the body point in I_* is $\mathbf{a}_* = \gamma_\tau^{-3} \mathbf{a}$ at body proper time τ . We take S as a rigid frame and therefore only neighboring points of S around the body are found to have the same acceleration. The frame S we call the Rindler frame. Consider also an infinite collection of inertial frames $\{I_\tau\}$ such that at body proper time τ , the body is located at point $(c^2/a, 0, 0)$ of I_τ . The I_τ frames have all axes parallel to those of I_* and their x -axes coincide with that of I_* . We set the proper time τ such that at $\tau = 0$ the corresponding I_τ coincides with I_* . So clearly $I_{\tau=0} = I_*$. If this is so then the hyperbolic motion guarantees that

$$x_* = \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), \quad (9)$$

$$t_* = \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right), \quad (10)$$

$$\beta_\tau = \frac{u_x(\tau)}{c} = \tanh\left(\frac{a\tau}{c}\right), \quad (11)$$

$$\gamma = (1 - \beta^2)^{-1/2} = \cosh\left(\frac{a\tau}{c}\right). \quad (12)$$

In what follows, we reproduce a brief sketch of the derivation of the electromagnetic contribution to the inertia reaction force in QED formulation [3,10,12]. It will be shown, as indicated in Eq. (34), that the final averaged results turn out to be the same for both developments (SED and QED).

The QED formulation of the zero-point electric and magnetic fields are given in Eq. (3) and Eq. (4). We now Lorentz transform these fields from the laboratory frame I_* into an instantaneously comoving frame I_τ to calculate the EM zero-point field vectors $\mathbf{E}_{z\tau}$ and $\mathbf{B}_{z\tau}$ of I_* but as represented in I_τ .

$$\begin{aligned} \mathbf{E}_\tau^{zp}(0, \tau) = & \sum_{\lambda=1}^2 \int d^3k H_{z\tau}(\omega) \left\{ \hat{x}\hat{\epsilon}_x + \hat{y} \cosh\left(\frac{a\tau}{c}\right) \left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\epsilon})_z \right] \right. \\ & \left. + \hat{z} \cosh\left(\frac{a\tau}{c}\right) \left[\hat{\epsilon}_z + \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\epsilon})_y \right] \right\} \left\{ \alpha(\mathbf{k}, \lambda) e^{i\Theta} + \alpha^\dagger(\mathbf{k}, \lambda) e^{-i\Theta} \right\}, \end{aligned} \quad (13)$$

$$\begin{aligned} \mathbf{B}_\tau^{zp}(0, \tau) = & \sum_{\lambda=1}^2 \int d^3k H_{z\tau}(\omega) \left\{ \hat{x}(\hat{k} \times \hat{\epsilon})_x + \hat{y} \cosh\left(\frac{a\tau}{c}\right) \left[(\hat{k} \times \hat{\epsilon})_y - \tanh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_z \right] \right. \\ & \left. + \hat{z} \cosh\left(\frac{a\tau}{c}\right) \left[(\hat{k} \times \hat{\epsilon})_z + \tanh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_y \right] \right\} \left\{ \alpha(\mathbf{k}, \lambda) e^{i\Theta} + \alpha^\dagger(\mathbf{k}, \lambda) e^{-i\Theta} \right\}, \end{aligned} \quad (14)$$

where Θ is given by

$$\Theta = \hat{k}_x \frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right) - \omega \frac{c}{a} \sinh\left(\frac{a\tau}{c}\right). \quad (15).$$

Here, unlike the classical random variable cases, the order of the quantum operator affects the results in the formulation, which is a major difference between the previous SED [3] and this present QED treatment.

We assume these fields as seen in I_τ to also correspond to the fields as *instantaneously* seen in S at proper time τ . Though the fields at the object point in S and in the corresponding point of the co-moving frame I_τ that instantaneously coincides with the object point are the same, this does not mean that detectors in S and in I_τ will experience the same radiation-field time evolution. A detector at rest in I_τ and the same detector at rest in S do not experience timewise the same effect. The two fields are the same at a given space-time point; however, the time evolution and space distribution of the field in S and those of the field in I_τ are not the same.

We consider next the ZPF radiation background of I_* in the act of, to put it graphically, being swept through by the object. Observe that this is not the ZPF of I_τ that in I_τ should be homogeneous and isotropic. For this we fix our attention on a fixed point of I_* , say the point of the observer at $(c^2/a, 0, 0)$ of I_* , that

momentarily coincides with the object at the object initial proper time $\tau = 0$, and consider that point as referred to another inertial frame I_τ that instantaneously will coincide with the object at a future generalized object proper time $\tau > 0$. Hence we compute the I_τ -frame Poynting vector, but as instantaneously evaluated at the $(c^2/a, 0, 0)$ space point of the I_* inertial frame, namely in I_τ at the I_τ space-time point:

$$ct_\tau = -\frac{c^2}{a} \sinh\left(\frac{a\tau}{c}\right), \quad x_\tau = -\frac{c^2}{a} \cosh\left(\frac{a\tau}{c}\right), \quad y_\tau = 0, \quad z_\tau = 0, \quad (16)$$

where the time in I_τ , called t_τ , is set to zero at the instant when S and I_τ (locally) coincide, which happens at proper time τ . (Here we correct a typo in Ref. [3], Eq. (20) where the minus sign in the RHS of the ct_τ equation in eqn. (16) does not appear.) Everything, however, is ultimately referred to the I_* inertial frame or laboratory frame. For further light on this point, see Appendix C of Ref. [3]. We first compute the ZPF Poynting vector that enters the body of the accelerating object in the instantaneous comoving frame I_τ ,

$$\begin{aligned} \mathbf{S}_*^{zp} &= \frac{c}{4\pi} \langle 0 | \mathbf{E}_\tau^{zp} \times \mathbf{B}_\tau^{zp} | 0 \rangle_* \\ &= \frac{c}{4\pi} \left\{ \hat{x} \langle 0 | E_y B_z - E_z B_y | 0 \rangle + \hat{y} \langle 0 | E_z B_x - E_x B_z | 0 \rangle + \hat{z} \langle 0 | E_x B_y - E_y B_x | 0 \rangle \right\}. \end{aligned} \quad (17)$$

The star in the equation above implies that the quantity needs to be evaluated in the laboratory inertial frame I_* . Since it turns out that only the two terms of the x -component of the ZPF Poynting vector are non-vanishing and the other seven components are zero, only these two non-vanishing terms will be examined here. For detailed calculations of all quantum averages, see Ref. [10]. Their values exactly match those of the SED analyses in Appendix A of Ref. [3].

In order to evaluate the vacuum expectation value $\langle 0 | E_y B_z | 0 \rangle$, the y -component of the electric field operators (13) and the z -component of the magnetic field operators (14), are multiplied together. The resulting expression has four terms, but as stated earlier only the term proportional to $\langle 0 | \alpha(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') | 0 \rangle$ remains as in (5a), and the expression simplifies to

$$\langle 0 | E_y B_z | 0 \rangle = \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \left[\cosh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_y - \sinh(\hat{k} \times \hat{\epsilon})_z \right] \left[\cosh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\epsilon})_z - \sinh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_y \right] \quad (18)$$

after one integration over the k -sphere. Each of the four terms in the equation above may be evaluated using the following polarization equations,

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y (\hat{k} \times \hat{\epsilon})_z = \hat{k}_x, \quad (19a)$$

$$\sum_{\lambda=1}^2 (\hat{k} \times \hat{\epsilon})_z (\hat{k} \times \hat{\epsilon})_z = \hat{k}_x^2 + \hat{k}_y^2 = 1 - \hat{k}_z^2, \quad (19b)$$

$$\sum_{\lambda=1}^2 \hat{\epsilon}_y^2 = 1 - \hat{k}_y^2, \quad (19c)$$

and the expression becomes

$$\langle 0 | E_y B_z | 0 \rangle = \sum_{\lambda=1}^2 \int d^3 k H_{zp}^2(\omega) \left\{ \left[\cosh^2\left(\frac{a\tau}{c}\right) + \sinh^2\left(\frac{a\tau}{c}\right) \right] \hat{k}_x - \cosh\left(\frac{a\tau}{c}\right) \sinh\left(\frac{a\tau}{c}\right) [1 + \hat{k}_x^2] \right\}. \quad (20)$$

Compare this to Eq. (A30) in Ref. [3]. The first term above is zero since $\int d^3 k \hat{k}_x = 0$. With the relation $\sinh \theta \cosh \theta = \frac{1}{2} \sinh(2\theta)$ and the change of variable from k to ω , the expression simplifies to

$$\langle 0|E_y B_z|0\rangle = -\frac{4\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \quad (21)$$

The other non-vanishing term $\langle 0|E_z B_y|0\rangle$ can also be evaluated following the same procedure as above and it is found that these two terms have the same magnitude but the opposite sign. With these results, the Poynting vector \mathbf{S}_*^{zp} of (17) becomes

$$\mathbf{S}_*^{zp} = -\hat{x} \frac{c}{4\pi} \frac{8\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega. \quad (22)$$

This represents the energy flux, i.e., the ZPF energy that enters the uniformly accelerating object's body per unit area per unit time from the viewpoint of the observer at rest in the inertial laboratory frame I_* . The radiation entering the body is that of the ZPF centered at the I_* frame of the observer. This leads to some detailed consideration on the so called k -sphere associated with each one of the inertial frames. For subtleties on this fine but important point, we refer to the Appendix C of Ref. [3].

The net impulse given by the field to the accelerated object, i.e., the total amount of momentum of the ZPF background the object has swept through after a time duration t_* , as judged again from the I_* -frame viewpoint, is

$$\mathbf{p}_* = \mathbf{g}_* V_* = \frac{\mathbf{S}_*}{c^2} V_* = -\hat{x} \frac{1}{c^2} \frac{c}{4\pi} \gamma_\tau^2 \beta_\tau \frac{2}{3} \langle \mathbf{E}_*^2 + \mathbf{B}_*^2 \rangle V_*. \quad (23)$$

Combining this with Eq.(11), (12) and (22) we obtain

$$\mathbf{S}_*(\tau) = \hat{x} \frac{c}{4\pi} \langle E_y B_z - E_z B_y \rangle = \hat{x} \frac{c}{4\pi} \frac{8\pi}{3} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \quad (24)$$

where as in Eqs. (21–23) the integration is understood to proceed over the k -sphere of I_* . This k -sphere is a subtler point referring to the need to regularize certain *prima facie* improper integrals. $\mathbf{S}_*(\tau)$ represents energy flux, and it also implies a parallel, x -directed momentum density, i.e., field momentum growth per unit time and per unit volume as it is incoming towards the object position, $(c^2/a, 0, 0)$ of S , at object proper time τ and as estimated from the viewpoint of I_* . Explicitly such momentum density is

$$\mathbf{g}_*^{zp}(\tau) = \frac{\mathbf{S}_*(\tau)}{c^2} = -\hat{x} \frac{8\pi}{3} \frac{1}{4\pi c} \sinh\left(\frac{2a\tau}{c}\right) \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega, \quad (25)$$

where we now introduce the henceforth frequency-dependent coupling or interaction coefficient $0 \leq \eta(\omega) \leq 1$, that quantifies the fractional amount of interaction at each frequency.

Let V_0 be the proper volume of the object. From the viewpoint of I_* , however, because of Lorentz contraction such volume is then $V_* = V_0/\gamma_\tau$. The amount of momentum due to the field inside the volume of the object according to I_* , i.e., the field momentum in the volume of the object viewed at the laboratory is

$$\mathbf{p}_*(\tau) = V_* \mathbf{g}_* = \frac{V_0}{\gamma_\tau} \mathbf{g}_*(\tau) = -\hat{x} \frac{4V_0}{3} c \beta_\tau \gamma_\tau \left(\frac{1}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \right), \quad (26)$$

which is again Eq. (23).

At proper time $\tau = 0$, the $(c^2/a, 0, 0)$ point of the laboratory inertial system I_* instantaneously coincides and comoves with the object point of the Rindler frame S in which the object is fixed. The observer located at $x_* = c^2/a$, $y_* = 0$, $z_* = 0$ instantaneously, at $t_* = 0$, coincides and comoves with the object but because the latter is accelerated with constant proper acceleration \mathbf{a} , the object according to I_* should receive a time rate of change of incoming ZPF momentum of the form:

$$\frac{d\mathbf{p}_*}{dt_*} = \frac{1}{\gamma_\tau} \left. \frac{d\mathbf{p}_*}{d\tau} \right|_{\tau=0} \quad (27)$$

We identify this expression with a force from the ZPF on the object. If the object has a proper volume V_0 , the force exerted on the object by the radiation from the ZPF as seen in I_* at $t_* = 0$ is then

$$\mathbf{f}_* = \frac{d\mathbf{p}_*}{dt_*} = -\left(\frac{4}{3} \frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega\right) \mathbf{a}. \quad (28)$$

Furthermore

$$m_i = \frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \quad (29)$$

is an invariant scalar with the dimension of mass. Observe that in Eq. (29) we have neglected a factor of $4/3$. A fully covariant analysis (See Appendix D of Ref. [3]) shows that it should be replaced by unity. We show this covariant analysis in QED formulation in our Appendix B. The corresponding form of m_i is then the mass of that fraction of the energy of the ZPF radiation enclosed within the object that interacts with the object as parametrized by the $\eta(\omega)$ factor in the integrand. Observe that $\eta(\omega) \rightarrow 0$ as $\omega \rightarrow \infty$ because all bodies become transparent at sufficiently high frequencies. For further discussions on these developments we refer to the already published literature [3,12,13,16,17].

4. Relativistic four-force expression of Newton's Second Law

This analysis yields not just the nonrelativistic Newtonian case but also a fully relativistic description within special relativity, at least for the case of longitudinal forces, i.e., forces parallel to the direction of motion. Moreover the extension to the more general case, where the accelerating or applied force, \mathbf{f} , is non-uniform, (i.e., it changes both in magnitude and direction throughout the motion of the object), has been in principle accomplished [3].

From the definition of the momentum \mathbf{p}_* in Eqs. (26, 29), it easily follows that the momentum of the body is $\mathbf{p}_* = m_i \gamma_\tau \beta_\tau c$, in agreement with the momentum expression in special relativity. The space 3-vector component of the four-force [14] is then

$$\mathbf{F}_* = \gamma_\tau \frac{d\mathbf{p}_*}{dt_*} = \frac{d\mathbf{p}_*}{d\tau}, \quad (30)$$

and as the force is pure in the sense of Rindler [14], the correct form for the four-force immediately follows,

$$\mathcal{F}_* = \frac{d\mathcal{P}}{d\tau} = \frac{d}{d\tau}(\gamma_\tau m_i c, \mathbf{p}) = \gamma_\tau \left(\frac{1}{c} \frac{dE}{dt}, \mathbf{f} \right) = \gamma_\tau (\mathbf{f} \cdot \beta_\tau, \mathbf{f}) = (\mathbf{F} \cdot \beta_\tau, \mathbf{F}). \quad (31)$$

Consistency with Special Relativity is established. A more detailed discussion leading to Eqs. (30)-(31) appears in Ref. [3], in particular in its Appendix D.

We evaluated the Poynting vector of the ZPF radiation field that an object under a constant proper acceleration (hyperbolic motion) sweeps through as seen from the laboratory frame I_* , and found that there appears to exist an interaction between the object under hyperbolic motion and the ZPF (inertia reaction force), whose magnitude is proportional to the acceleration, implying that the ZPF possess a structure which reacts against acceleration. We propose that this reaction force between the accelerated object and the ZPF background radiation is a part of what we know as inertia.

Appendix A: Correspondence between SED and QED

In this appendix, it is shown why the Poynting vector, $\mathbf{S} = \frac{c}{4\pi}(\mathbf{E} \times \mathbf{B})$, indeed gives identical results for SED and QED averagings. For this purpose, let us see the case of $\langle 0|E_y B_z|0\rangle$, one of the two non-vanishing terms. The other seven terms happen to vanish both in QED formulations [10] and in SED [3].

To evaluate $\langle 0|E_y B_z|0\rangle$, we multiply the y -component of ZPF electric field and the z -component of the magnetic field as given in Eqns. (14,15) to obtain

$$\langle 0|E_y B_z|0\rangle = \sum_{\lambda=1}^2 \sum_{\lambda'=1}^2 \int d^3 k \int d^3 k' \sqrt{\frac{\hbar\omega}{2\pi^2}} \sqrt{\frac{\hbar\omega'}{2\pi^2}}$$

$$\begin{aligned}
& \times \cosh^2\left(\frac{a\tau}{c}\right) \left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\epsilon})_z \right] \left[(\hat{k} \times \hat{\epsilon})_z - \tanh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_y \right] \\
& \times \frac{1}{2} \langle 0 | [\alpha(\mathbf{k}, \lambda) e^{i\Theta} + \alpha^\dagger(\mathbf{k}, \lambda) e^{-i\Theta}] [\alpha(\mathbf{k}', \lambda') e^{i\Theta'} + \alpha^\dagger(\mathbf{k}', \lambda') e^{-i\Theta'}] | 0 \rangle. \quad (32a)
\end{aligned}$$

This equation has four terms. However, only the term proportional to $\langle 0 | \alpha(\mathbf{k}, \lambda) \alpha^\dagger(\mathbf{k}', \lambda') | 0 \rangle$ remains non-vanishing due to the vacuum expectation values given by Eqns. (5a, 5b). Thus, after an intergration over the k -sphere, the right hand side of Eq. (32a) becomes

$$\langle 0 | E_y B_z | 0 \rangle = \frac{1}{2} \int d^3 k \frac{\hbar \omega}{2\pi^2} \cosh^2\left(\frac{a\tau}{c}\right) \left[\hat{\epsilon}_y - \tanh\left(\frac{a\tau}{c}\right) (\hat{k} \times \hat{\epsilon})_z \right] \left[(\hat{k} \times \hat{\epsilon})_z - \tanh\left(\frac{a\tau}{c}\right) \hat{\epsilon}_y \right]. \quad (32b)$$

This expression can be evaluated with the help of the polarization relations, Eq. (19), and the angular integration $\int \hat{k}_x^2 d\Omega = \int \sin^3 \theta d\theta \int \cos^2 \phi d\phi = 4\pi/3$, and we find that

$$\langle 0 | E_y B_z | 0 \rangle = -\frac{4\pi}{c} \sinh\left(\frac{2a\tau}{c}\right) \int \frac{\hbar \omega^3}{2\pi^2 c^3} d\omega, \quad (33)$$

which is the same value as the SED analogue $\langle E_y B_z \rangle$, as already reported in Appendix A of Ref. [3]. It can be shown easily, following the same procedures, that the other non-vanishing term $\langle 0 | E_z B_y | 0 \rangle$ also yields exactly the same value as the SED case. Thus, the correspondence between SED and QED is achieved, and we can indeed write

$$\langle \mathbf{S} \rangle = \frac{c}{4\pi} \langle \mathbf{E} \times \mathbf{B} \rangle = \frac{c}{4\pi} \langle 0 | \overline{\mathbf{E}} \times \overline{\mathbf{B}} | 0 \rangle = \langle 0 | \mathbf{S} | 0 \rangle \quad (34)$$

Appendix B: Covariant Approach

In this section, the electromagnetic ZPF Poynting vector $\mathbf{S}^{zp} = \frac{c}{4\pi} (\mathbf{E}^{zp} \times \mathbf{B}^{zp})$ and its vacuum expectation values are to be evaluated using a covariant method. It will be shown, following the approach by Rohrlich [15], and Appendix D of Ref. [3], that the factor of 4/3 for an expression of inertial mass, obtained earlier in the non-covariant method, vanishes in this fully covariant approach.

The Poynting vector \mathbf{S} is an element of the symmetrical *electromagnetic energy-momentum tensor*

$$\Theta^{\mu\nu} = \begin{pmatrix} -U & -S_x/c & -S_y/c & -S_z/c \\ -S_x/c & T_{xx} & T_{xy} & T_{xz} \\ -S_y/c & T_{yx} & T_{yy} & T_{yz} \\ -S_z/c & T_{zx} & T_{zy} & T_{zz} \end{pmatrix} \quad (35)$$

In the above, the time and mixed space-time components are

$$\Theta^{00} = \frac{1}{8\pi} (E^2 + B^2) \equiv -U' \text{ and } \Theta^{0i} = -\frac{1}{4\pi} (\mathbf{E} \times \mathbf{B})_i \quad (36)$$

where U is the electromagnetic energy density and S is the *Poynting vector*. The space part of the tensor Θ^{ij} is the *Maxwell stress tensor* whose components are given as

$$T_{ij} = \frac{1}{4\pi} \left[E_i E_j + B_i B_j - \frac{1}{2} (E^2 + B^2) \delta_{ij} \right] \quad (37)$$

Now let us consider the quantity,

$$P^\mu \equiv \frac{1}{c} \int \Theta d\sigma_\nu = \left(\frac{1}{c} W, \mathbf{P} \right) \quad (38)$$

the integration of the energy-momentum tensor over a spacelike plane σ given by the equation $n^\mu x_\mu + c\tau = 0$, where n^μ is the unit normal vector to the three dimensional hyperplane. Any instant of an inertial observer is characterized by this spacelike plane σ and the unit normal n^μ . For example, when $n^\mu = (1; 0, 0, 0)$, $\tau = t$,

then the spacelike plane σ describes the xyz -plane at the instant t . For further details on this point, we refer the reader to Ref. [15] and Appendix D of Ref. [3].

In the particular Lorentz frame whose surface normal is given by $n^\nu = (1; 0, 0, 0)$, the components of P^μ can be given explicitly as

$$W^{(0)} = \int U^{(0)} d^3x, \text{ and, } \mathbf{P}^{(0)} = \frac{1}{c^2} \int \mathbf{S}^{(0)} d^3x. \quad (39)$$

However, in the case of interest to us in which the velocity is along the positive x -direction, the surface normal is given by $n^\nu = (\gamma; \gamma\beta\hat{\mathbf{n}})$, and Eq. (38) takes the following forms:

$$W = \gamma \int U d\sigma - \frac{\gamma\beta}{c} \int \mathbf{S} \cdot \hat{\mathbf{n}} d\sigma, \text{ and, } \mathbf{P} = \frac{\gamma}{c^2} \int \mathbf{S} d\sigma + \frac{\gamma\beta}{c} \int \overleftrightarrow{\mathbf{T}} \cdot \hat{\mathbf{n}} d\sigma. \quad (40)$$

At this point, we identify P^μ of Eq. (38) as the momentum four-vector of the electromagnetic field. Note in passing that extra terms appear in (40), which can also be obtained from the corresponding Lorentz transformation. Abraham and Lorentz used the Eqns. (39) as their definitions for the energy density and the momentum in the case of the Coulomb self-field of the classical electron, and they were led to the incorrect factor of $4/3$ for the momentum of an electron. However, Eqns. (39) are only valid in the particular Lorentz frame where γ is 1, when the second terms in Eqns. (40) vanish. We show that with the use of the correct forms Eqns. (40) for the energy density and the momentum, this incorrect factor of $4/3$ is reduced to unity, as should be expected.

The expressions that we need to evaluate are

$$P^0 = \frac{\gamma}{c} \int (U - \mathbf{v} \cdot \mathbf{g}) d^3\sigma, \text{ and, } \mathbf{p}_* = \gamma \left(\mathbf{g}_* + \frac{\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}_*}{c^2} \right) V_0 \quad (41)$$

where the latter is the momentum of the background ZPF the object has swept through as seen from the lab inertial frame I_* . The dot product of $\overleftrightarrow{\mathbf{T}}_*$ with the velocity $\mathbf{v} = v\hat{x}$ in the above equation yields the column vector $\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v} = (\hat{x}T_{xx} + \hat{y}T_{yx} + \hat{z}T_{zx})v$ with T_{ij*} given by (37) and $\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, etc. It turns out that only

the x -component has non-zero value, and the y and z components of the expectation values in $\overleftrightarrow{\mathbf{T}} \cdot \mathbf{v}$ vanish, which is physically reasonable since the object is moving in the positive x -direction. For the x -component, we have

$$\begin{aligned} \langle 0|T_{xx*}|0\rangle &= \frac{1}{4\pi} \langle 0|E_{x\tau}E_{x\tau} + B_{x\tau}B_{x\tau} - \frac{1}{2}(E_\tau^2 + B_\tau^2)|0\rangle_* \\ &= \frac{1}{4\pi} \langle 0|E_{x\tau}^2 + B_{x\tau}^2|0\rangle_* - \frac{1}{8\pi} \langle 0|E_\tau^2 + B_\tau^2|0\rangle_* \end{aligned} \quad (42)$$

where

$$E_\tau^2 = E_{x\tau}^2 + E_{y\tau}^2 + E_{z\tau}^2, \text{ and } B_\tau^2 = B_{x\tau}^2 + B_{y\tau}^2 + B_{z\tau}^2. \quad (43)$$

The first term becomes

$$\begin{aligned} \frac{1}{4\pi} \langle 0|E_{x\tau}^2 + B_{x\tau}^2|0\rangle_* &= \frac{1}{4\pi} \langle 0|E_{x*}^2 + B_{x*}^2|0\rangle \\ &= \frac{1}{12\pi} \langle 0|E_*^2 + B_*^2|0\rangle, \end{aligned} \quad (44)$$

considering equal contributions from each direction. After the substitution of

$$U = \frac{1}{8\pi} \langle 0|E_*^2 + B_*^2|0\rangle = \int \frac{\hbar\omega^3}{2\pi^2c^3} d\omega \quad (45)$$

we find that for the first term of (42),

$$\frac{1}{4\pi}\langle 0|E_{x\tau}^2 + B_{x\tau}^2|0\rangle_* = \frac{2}{3}\int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega. \quad (46)$$

For the evaluation of the second term of (42), we find the Lorentz transformed field components from Eqns. (13,14), and notice that the squared fields have contributions given by Eq. (45) to obtain

$$\langle 0|T_{xx*}|0\rangle = \frac{1}{3}\int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega(1 - 2\gamma_\tau^2 - 2\gamma_\tau^2\beta_\tau^2). \quad (47)$$

Using the two results above, we can obtain for the momentum

$$\mathbf{p}_* = \gamma_\tau\left(\mathbf{g}_* + \frac{\overleftrightarrow{\mathbf{T}}_* \cdot \mathbf{v}_*}{c^2}\right)V_0 = \hat{x}\gamma_\tau V_0 c\beta_\tau \frac{1}{c^2}\int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega. \quad (48)$$

We note here that the extra factor of 4/3 obtained earlier in a non-covariant method becomes unity, as expected, in this covariant approach.

Following similar steps, we can also evaluate the zero-component of the momentum four-vector as

$$P^0 = \frac{\gamma_\tau}{c}\left[\frac{\langle 0|E_*^2 + B_*^2|0\rangle}{8\pi} - c\beta_\tau g_*\right]V_0 + \frac{\gamma_\tau V_0}{c}\int \frac{\hbar\omega^3}{2\pi^2c^3}d\omega. \quad (49)$$

The inertia reaction force that is exerted upon the object by the ZPF is

$$\begin{aligned} \mathbf{f}_*^{zp} &= -\frac{d\mathbf{p}_*}{dt_*} = -\frac{1}{\gamma_\tau}\frac{d\mathbf{p}}{dt_*} \\ &= -\left(\frac{V_0}{c^2}\int \eta(\omega)\frac{\hbar\omega^3}{2\pi^2c^3}d\omega\right)\mathbf{a} \end{aligned} \quad (50)$$

With the identification of the quantity inside the parenthesis as the inertial mass m_i , we can obtain the four-force as

$$F^\mu = \frac{dP^\mu}{d\tau} = \frac{d}{d\tau}(m_i c\gamma_\tau; \mathbf{p}) = \gamma_\tau\left(\frac{1}{c}\frac{dE}{dt}; \frac{d\mathbf{p}}{dt}\right) = \gamma_\tau(\mathbf{f} \cdot \beta_\tau; \mathbf{f}) = \gamma_\tau(\mathbf{F} \cdot \beta_\tau; \mathbf{F}) \quad (51)$$

which is the same expression as the Eq. (31) above.

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